1/2/15. The faces of 27 unit cubes are painted red, white, and blue in such a manner that we can assemble them into three different configurations: a red $3 \times 3 \times 3$ cube, a white $3 \times 3 \times 3$ cube, and a blue $3 \times 3 \times 3$ cube. Determine, with proof, the number of unit cubes on whose faces all three colors appear.

2/2/15. For any positive integer $n$, let $s(n)$ denote the sum of the digits of $n$ in base 10. Find, with proof, the largest $n$ for which $n = 7s(n)$.

3/2/15. How many circles in the plane contain at least three of the nine points $(0,0)$, $(0,1)$, $(0,2)$, $(1,0)$, $(1,1)$, $(1,2)$, $(2,0)$, $(2,1)$, $(2,2)$? Rigorously verify that no circle was skipped or counted more than once in the result.

4/2/15. In how many ways can one choose three angle sizes, $\alpha$, $\beta$, and $\gamma$, with $\alpha \leq \beta \leq \gamma$ from the set of integral degrees, $1^\circ$, $2^\circ$, $3^\circ$, $\ldots$, $178^\circ$, such that those angle sizes correspond to the angles of a nondegenerate triangle? How many of the resulting triangles are acute, right, and obtuse, respectively?

5/2/15. Clearly draw or describe a convex polyhedron that has exactly three pentagons among its faces and the fewest edges possible. Prove that the number of edges is a minimum.

Complete, well-written solutions to at least two of the problems above, accompanied by a Cover Sheet, should be mailed to:

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279 East Central St Suite 246
Franklin, MA 02038-1317

and postmarked by Sunday, 23 November 2003. Each participant is expected to develop solutions without help from others. For the cover sheet and other details, see the USAMTS web site: http://www.nsa.gov/programs/mepp/usamts.html.