1/2/11. The number N consists of 1999 digits such that if each pair of consecutive digits in N were viewed as a two-digit number, then that number would either be a multiple of 17 or a multiple of 23. The sum of the digits of N is 9599. Determine the rightmost ten digits of N.

2/2/11. Let C be the set of non-negative integers which can be expressed as \(1999s + 2000t\), where \(s\) and \(t\) are also non-negative integers.
(a) Show that 3,994,001 is not in C.
(b) Show that if \(0 \leq n \leq 3,994,001\) and \(n\) is an integer not in C, then 3,994,001 - \(n\) is in C.

3/2/11. The figure on the right shows the map of Squareville, where each city block is of the same length. Two friends, Alexandra and Brianna, live at the corners marked by A and B, respectively. They start walking toward each other’s house, leaving at the same time, walking with the same speed, and independently choosing a path to the other’s house with uniform distribution out of all possible minimum-distance paths [that is, all minimum-distance paths are equally likely]. What is the probability they will meet?

4/2/11. In \(\triangle PQR\), \(PQ = 8\), \(QR = 13\), and \(RP = 15\). Prove that there is a point S on line segment \(\overline{PR}\), but not at its endpoints, such that PS and QS are also integers.

5/2/11. In \(\triangle ABC\), \(AC > BC\), CM is the median, and CH is the altitude emanating from C, as shown in the figure on the right. Determine the measure of \(\angle MCH\) if \(\angle ACM\) and \(\angle BCH\) each have measure 17°.

Complete, well-written solutions to at least two of the problems above, accompanied by a completed Cover Sheet should be sent to the following address and postmarked no later than November 15, 1999. Each participant is expected to develop solutions without help from others.

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