USA Mathematical Talent Search
Round 3 Grading Rubric
Year 34 — Academic Year 2022–2023
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GENERAL GUIDELINES

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.

2. On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

4. A student's justification needs to be rigorous and reasonably clear in order for the solution to earn **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, deduct points accordingly.

Problem 1/3/34:

Award **5 points** for the correct configuration of X's and Y's. Withhold **1 point** for each X or Y that is filled in incorrectly, or for each extra or missing X or Y.

Problem 2/3/34:

Note: There are a variety of correct answers to this problem. Any function that works and is accompanied by sufficient explanation should receive **5 points**.

Award **1 point** for a function that works and **4 points** for explaining why the function works. Withhold points as appropriate for significant gaps in the explanation. If a student has an incorrect function that contains significant constructive progress towards a correct function, graders have the discretion to award partial credit for correct and useful ideas.

Note: One theme in correct answers was that they involved partitioning the positive integers in some way. For example, the official solution partitions the integers based on the value of $\nu_p(n) \pmod 5$. We awarded **4 points** for a defective partition with a trivial error, and a maximum of **2 points** for a badly constructed partition.

Problem 3/3/34:

Note: Solutions that were similar to the official solution were scored as follows:

1 point: Student recognizes that it is useful to consider $N = qx + r$, and student manipulates the given equation accordingly.

1 point: Student correctly analyzes the case in which $x \le \lfloor \sqrt{N} \rfloor$ (and $q \ge \lfloor \sqrt{N} \rfloor$).

1 point: Student correctly analyzes the case in which $x \ge \lfloor \sqrt{N} \rfloor$ (and $q \le \lfloor \sqrt{N} \rfloor$).

1 point: Student correctly analyzes the overlap between the above cases.

1 point: Student uses the fact that there are 10^{100} solutions to determine the bounds on N.

Note: An alternative method was scored as follows:

2 points: Student recognizes and proves that x satisfies the equation if $\lfloor \frac{N}{x} \rfloor > \lfloor \frac{N}{x+1} \rfloor$ and $1 \le x \le N$ (**1 point** for stating this result, **1 point** proving this result).

1 point: Student shows that for $x \le \sqrt{N}$, the values of $\lfloor \frac{N}{x} \rfloor$ are all different.

1 point: Student shows that for $x \ge \sqrt{N}$, every value from 1 to $\lfloor \frac{N}{\sqrt{N}} \rfloor$ occurs.

1 point: Student shows that we have a total of $2 \cdot \lfloor \sqrt{N} \rfloor - 1$ values if $\lfloor \frac{N}{\sqrt{N}} \rfloor = \lfloor \sqrt{N} \rfloor$, and otherwise $2 \cdot \lfloor \sqrt{N} \rfloor$ values, and arrives at the correct answer.

Note: For either solution method, a mostly correct solution with the wrong total in one case usually received **3 points**, losing **1 point** for the case and **1 point** for the wrong answer.

Note: Award **1 point** for guessing a pattern to get the correct range of integers with no proof. A total score of **2 points** may be awarded if there is a meaningful result that would be part of a proof.

Note: Award **5 points** for a count of the number of googolicious numbers if the correct set is given in the proof.

Problem 4/3/34:

3 points: Student shows that the region of snug points is an ellipse. Award partial credit for significant constructive progress, such as using the lemma in the official solution (**1 point**), and **1 point** for an additional significant result.

1 point: Student shows that any point outside the ellipse is not snug.

1 point: Student correctly computes the area of the ellipse.

Problem 5/3/34:

Note: In retrospect, it would have been better to specify explicitly that we were asking whether there exists a **non-empty** finite set S with the desired properties. Nonetheless, we believe it was sufficiently clear that this was the intent of the problem. As the empty set is a trivial solution, we did not award any credit for saying that the empty set satisfies the criteria in the problem.

Note: Solutions that were similar to the official solution were scored as follows:

1 point: Student finds a correct construction in terms of coordinates.

4 points: Student shows that the construction meets the criteria for each of the four slopes (**1 point** each).

Note: An additional valid approach was to use Cauchy induction. These solutions were scored as follows:

1 point: Student explains a plan to use Cauchy induction, including what this entails.

1 point: Student proves the problem for $n = 2$.

1 point: Student proves that the result for $n = k$ implies the result for $n = 2k$.

2 points: Student proves that the result for $n = k$ implies the result for $n = k - 1$.

Note: Across all solution methods, we awarded partial credit for the following constructive progress.

1 point: Student solves the problem for a smaller value, such as 2 instead of 2022.

1 point: Student extends their work to a higher number of intersection points.