USA Mathematical Talent Search
Round 2 Grading Rubric
Year 34 — Academic Year 2022–2023
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GENERAL GUIDELINES

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.

2. On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

4. A student's justification needs to be rigorous and reasonably clear in order for the solution to earn **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, deduct points accordingly.

Problem 1/2/34:

Award **5 points** for the correct configuration of numbers. No justification is required. Withhold **1 point** for each square that is filled in incorrectly.

Problem 2/2/34:

3 points: Student finds the correct expected value expression. Award partial credit for constructive progress as appropriate. Finding the expected value for the first "thing" Grogg does is conceptually important enough to earn **1 point**.

2 points: Student finds p in terms of n or vice versa. Award **1 point** for significant constructive progress, such as the factorization.

Note: Some students divided by $1 - p$ without noting that $p \neq 1$. We didn't deduct points for this, but it is helpful to note that $p \neq 1$ means we won't be dividing by 0.

Problem 3/2/34:

1 point: Student obtains the correct answer that Lizzie can guarantee a win if and only if n is composite. Award this point even if the student provides no explanation.

2 points: Student proves that Lizzie can always win if n is composite. Award **1 point** for describing Lizzie's strategy and **1 point** for showing how it leads Lizzie to victory.

2 points: Student proves that Lizzie might not be able to win if n is prime. Award **1 point** for describing a counterexample in terms of the numbers Alex picks, and award **1 point** for explaining why Lizzie will be unable to win.

Note: For the counterexample, some students used $n-1$ zeroes and 1 one. This violates the condition that the numbers are positive. We did not deduct a point for this error.

Note: Some students incorrectly divided the cases into even and odd rather than prime and composite. We awarded **2 points** for Lizzie's winning strategy if it clearly generalizes to composite n, or **1 point** if it's not clear that the strategy generalizes. Additionally, we awarded **1 point** for a counterexample that would be a counterexample for prime n.

Note: We awarded **3 points** for an otherwise-correct solution that incorrectly assumes that all of Lizzie's groups are size 3 (and thus she wins only if n is a power of 3).

Problem 4/2/34:

1 point: Student shows that if k is even, then $c_k = 4$.

Note: We awarded **1 point** for 4-coloring the plane with a construction that is valid for $k = 2$.

1 point: Student shows that if $k = 3$, then $c_k = 9$.

3 points: Student shows that if $k > 3$ is odd, then $c_k = 5$. Award partial credit as appropriate for significant constructive progress, including the situations described below.

Note: Award **2 of 3 points** for the case of odd $k > 3$ for a correct construction with $c_5 = 5$ extended to $c_7 = 5$ if it could be extended further.

Note: Award **1 of 3 points** for the case of odd $k > 3$ for a correct construction with $c_k = 6, 7$, or 8. Award only a total of **1 point** for odd $k > 3$, and thus a maximum score of **3 points**, even if there is also a construction for $k = 5$ giving $c_5 = 5$, unless significant progress is made towards extending this.

Note: Award **0 of 3 points** for the case of odd $k > 3$ for a correct construction with $c_k = 9$ or larger.

Note: Award a total score of **1 point** if the student writes the correct answer (including the values of c_k for all three cases), but doesn't provide any explanation.

Problem 5/2/34:

Note: The rubric is based on the official solution, which uses the properties of cyclic quadrilaterals. Solutions using other methods tended to be graded more holistically.

1 point: Student recognizes that thinking about cyclic quadrilaterals can be helpful, and obtains any meaningful result.

1 point: Student proves that C, D, F, G are the vertices of a cyclic quadrilateral.

1 point: Student uses the fact that C, D, F, G are the vertices of a cyclic quadrilateral to obtain any further meaningful result.

1 point: Student proves that $CDPG$ is an isosceles trapezoid.

1 point: Student computes the vertices of P.

Note: If the solution has a good diagram, the solution should receive at least **1 point**. If the solution is missing a diagram and the absence of a diagram significantly hinders comprehension, deduct **1 point.**.

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Please visit **http://www.usamts.org** for details about solution submission.