USA Mathematical Talent Search

Round 3 Grading Rubric

Year 30 — Academic Year 2018–2019

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General Guidelines

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, graders consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.
2. On **all** problems, graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.
3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.
4. A student's justification needs to be rigorous and reasonably clear in order for the solution to merit **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, graders deduct points accordingly.

Problem 1/3/30:

Award **5 points** for the correct configuration of numbers. No justification is required. Withhold **1 point** for each incorrect entry.

Problem 2/3/30:

1 point: Student shows that there is a relationship between the values of p_k and Pascal's triangle (or $\binom{n}{r}$).

1 point: Student recognizes that if t is the $(m-1)$st triangular number, then $p_{t+i} = \binom{m}{i}$ for $1 \le i \le m$.

2 points: Student recognizes that $\binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{n} = 2^n - 1$ and applies this to the terms in $p_1 + p_2 + \cdots + p_{499500}$ to obtain the sum $(2-1)+(2^2-1)+(2^3-1)+\cdots+(2^{999}-1)$. Award **1 point** for significant constructive progress towards this result.

1 point: Student recognizes that $(2 + 2^2 + \cdots + 2^{999}) = 2^{1000} - 2$, and arrives at the correct answer of $2^{1000} - 1001$.

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Note: If the student writes the correct answer without explanation, award **1 point**.

Problem 3/3/30:

Submissions similar to the official solution were scored as follows:

1 point: Student applies Ptolemy's Theorem to determine that $ac + bd = 2K$, where K is the area of $ABCD$.

1 point: Student determines that $bd - ac = \frac{25}{2}$.

1 point: Student recognizes that the area of $ABCD$ is maximized when $b^2 + d^2$ is minimized.

1 point: Student applies the Cauchy-Schwarz Inequality to obtain $b^2 + d^2 \geq \frac{169}{2}$.

1 point: Student determines that maximum area of $ABCD$ is 36, with or without explanation.

Note: If the student did not say anything about how an area of 36 is attainable, the solution received at most **4 points**.

Note: If the student obtained a higher upper bound (usually by not considering some of the constraints in the problem), the solution received at most **3 points**.

Note: If the student did not include any discussion of inequalities or maximization, the solution received at most **2 points**.

Problem 4/3/30:

Students must do two things to achieve a complete solution. First, they must show that it is possible for 999,998 squares to be covered by eels **(3 points)**. Second, they must show that it is impossible to cover more than 999,998 squares with eels **(2 points)**.

3 points: Student shows that 999,998 squares can be covered by eels, and explains their construction clearly. Award **2 points** of partial credit for a clear construction that shows that 999,996 or 999,997 squares can be covered by eels. Award **1 point** of partial credit for a non-trivial lower bound (i.e., $\geq$ 999,500 covered squares).

2 points: Student shows that it is impossible to cover more than 999,998 squares with eels. The result that the bottom row must have an uncovered square is worth **1 point** of partial credit.

Problem 5/3/30:

Submissions similar to the official solution were scored as follows:

1 point: Student develops a **useful connection** to modular arithmetic. In other words, to receive this point, students had to do something beyond just having the insight that using modular arithmetic might be a good strategy. In particular, it wasn't sufficient to have a few expressions involving modular congruences that were reasonable thoughts, but didn't actually lead in the direction of a valid solution. The solution needed to be headed in the right direction in some meaningful way in order to receive any credit.

1 point: Student recognizes that the number of residue classes is $\lfloor p/4 \rfloor + 1$.

1 point: Student recognizes that if no $x_i = 0$ for $0 \le i \le m$, then no x_i in the infinite sequence $x_0, x_1, x_2, \ldots$ is equal to 0.

1 point: Student recognizes that $k \le m = \lfloor p/4 \rfloor + 1$.

1 point: Student completes the analysis by examining what happens if p is odd and if $p = 2$.

Note: Solutions that did not discuss the case of $p = 2$ received at most **4 points**.

Note: Showing that all the a_n are relatively prime was worth **1 point**.

Note: Any significant result was worth **1 point**, and in some cases, the grading team made judgment calls based on how far or close the student was to a complete solution.