USA Mathematical Talent Search

Round 1 Grading Rubric

Year 30 — Academic Year 2018–2019

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GENERAL GUIDELINES

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.

2. On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

4. A student's justification needs to be rigorous and reasonably clear in order for the solution to merit **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, deduct points accordingly.

Problem 1/1/30:

Award **5 points** for the correct configuration of numbers. No justification is required. Withhold **1 point** for each incorrect entry.

Problem 2/1/30:

1 point: Student recognizes that all orangutoads in the leftmost position must move 1 unit to the right.

1 point: Student recognizes that all orangutoads in the rightmost position must move 1 unit to the left.

Note: If the student doesn't provide justification for the above claims, only award **1 point** out of the above **2 points**.

1 point: Student recognizes that after each round, the outermost orangutoads are closer.

2 points: Student explains why the fact that the outermost orangutoads get closer after each round means that some orangutoad will eventually be unable to move.

Note: If the student assumes that orangutoad k starts at position k for all $1 \le k \le n$ or assumes that orangutoad 1 is the leftmost orangutoad, award at most **4 points**.

Note: Students do not need to formalize their argument algebraically to get **5 points** on this problem, as long as their underlying reasoning is clear, rigorous, and correct.

Note: Some students solved the problem for a specific value of n. Award at most **1 point**, since the student isn't considering how their argument generalizes. Only award **1 point** if $n \ge 3$ and the student makes meaningful constructive progress that addresses one of the key ideas discussed above.

Problem 3/1/30:

Note: There were two valid solutions methods: (1) the casework method explained in the official solution and (2) recognizing that the largest sum that can be achieved with $a_1 = a$ must be one less than the smallest sum that can be obtained with $a_1 = a + 1$.

2 points: Student recognizes that $(n, d) = (3, 2)$ satisfies the conditions in the problem. Award **1 point** for the answer with at least some justification and **1 point** for a rigorous explanation that $(3, 2)$ works. Students who get an incorrect answer should not be penalized twice: once for the incorrect answer and once for the flaw in the reasoning.

3 points: Student proves that no other combinations of n and d satisfy the conditions in the problem. To do this, students must show five things:

(1): $n \neq 1$.

(2) $n \neq 2$.

(3) $n \ngtr 3$.

(4) $d \neq 1$ (for $n = 3$).

(5) $d \ngtr 2$ (for $n = 3$).

All five are required to receive **3 points**. Any three out of five are needed for **2 points**. Showing one of (2) through (5) is needed for **1 point**; (1) is sufficiently easy that it is insufficient for receiving any of these points.

Note: For students who set up the cases differently, award partial credit as appropriate based on the above guidelines.

Note: For students who missed the *unique* sequence criterion (e.g., obtained $n \geq 3$ and $d \geq 2$), award a maximum of **3 points**. This was a common error with the second solution method.

Problem 4/1/30:

Note: Student solution methods tended to be pretty variable. Solutions similar to the official solution were graded using the rubric below, but in many cases we determined the score based on how close (or far) a student submission was to a complete solution. We awarded **1 point** of partial credit for any meaningful result, and additional credit for further constructive progress.

1 point: Student recognizes that the path traced by the fly creates similar triangles.

1 point: Student uses similar triangles to obtain a useful result (e.g., ϕ is a dilation centered at A with ratio $\frac{AB}{AD} > 1$).

3 points: Student completes the proof using a proof by contradiction or other valid solution method. Award **1 point** of partial credit for any additional meaningful result and award **2 points** of partial credit if the student's proof is almost complete, but there is a non-trivial gap or a key claim that is not justified.

Problem 5/1/30:

Note: Students who write computer programs need to compute the exact numerical answer. Students who do NOT write computer programs do NOT need to compute the exact numerical answer, and instead can leave the answer in a reasonable form similar to the one in the official solution.

Note: If a student writes a computer program and provides sufficient documentation (i.e., their code) and obtains the correct answer, award **5 points**. If the student writes a computer program and does NOT obtain the correct answer, award partial credit if and only if the student demonstrates some of the key mathematical ideas needed to solve the problem in their code.

Note: Every so often, a student submits a computer program that can obtain the correct answer eventually, but the program is still running as of the submission deadline because the program is inefficient. Grade these solutions similarly to the programs in which the student obtains an incorrect answer (i.e., only award partial credit for key mathematical ideas in the student's code).

1 point: Student recognizes that $f(2n) = f(2n+1)$ if $2n$ is uphill and $f(2n) = f(2n+1) + 1$ if $2n$ is NOT uphill.

1 point: Student recognizes that it is sufficient to count the number of non-uphill even integers from 2 to $10^{2018} - 2$ inclusive and multiply this number by -1.

Note: If a student accomplished both of the above items, but misinterpreted the definition of uphill, we only awarded **1 point**.

3 points: Student successfully counts the number of uphill even integers (or uses another valid approach) and arrives at the correct answer. Award **1 point** of partial credit if the student proposes a viable way to count the number of uphill even integers and makes meaningful constructive progress towards completing the counting. Award **2 points** of partial credit if the student's proof is almost complete, but there is a non-trivial gap or a key claim that is not justified.

Note: We deducted **1 point** from the total score if the student's answer was $-1 \cdot$ Number of uphill even integers instead of $-1 \cdot$ Number of non-uphill even integers.

Note: We awarded a total of **3 points** for a correct recursive formula that could be solved in a feasible amount of work, such as $2018 \cdot 10$ computations, and **5 points** if the recursion was computed to give the right answer.

Note: We awarded a total of **2 points** for computing the first several terms, then fitting this to a function (usually by looking up $g(n) - 10^{\frac{n}{2}}$ in the Online Encyclopedia of Integer Sequences) and getting the correct formula to compute the value numerically. We did not award any points for an extrapolation from the wrong number of terms (such as fitting the first three terms to a quadratic), or for incorrect terms, unless there was something else to justify credit.

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