# USA Mathematical Talent Search

Round 2 Grading Rubric

Year 30 — Academic Year 2018–2019

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**General Guidelines**

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, graders consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.
2. On **all** problems, graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.
3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.
4. A student's justification needs to be rigorous and reasonably clear in order for the solution to merit **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, graders deduct points accordingly.

**Problem 1/2/30:**

Award **5 points** for the correct configuration of numbers/regions. No justification is required. Withhold **1 point** for each square that is in an incorrect region.

**Note:** Some students used color in their solutions, which made it hard to see the region boundaries. In these cases, we did not deduct any points for the submission being hard to follow. But please be mindful for the future that we print submissions in black and white, so students should NOT use color in their submissions.

**Problem 2/2/30:**

**1 point:** Student assumes for the sake of contradiction that $S_1 \neq S_2$. To get this point, it is NOT sufficient to propose a proof by contradiction without making a reasonable assumption for the sake of contradiction.

**1 point:** Student lets $n$ be the smallest positive integer that lies in exactly one of $S_1$ and $S_2$ (or does something equivalent). For illustrative purposes here, let $n$ lie in $S_1$, but not $S_2$.

**1 point:** Student recognizes that some proper divisor of $n$ (let's call this $d$) must lie in $S_2$.

**1 point:** Student recognizes that $d$ must also lie in $S_1$.

**1 point:** Student recognizes that the fact that both $d$ and $n$ lie in $S_1$ contradicts the condition that $S_1$ is an unfriendly set, so we can conclude that $S_1 = S_2$.

**Note:** Some students incorrectly assumed that the sets are necessarily finite. In this case, graders deducted **1 point** if (and only if) the student's argument as written did NOT also apply to infinite sets.

**Problem 3/2/30:**

**Note:** Some students used the elegant approach in the official solution, and other students used messier approaches. The guidelines below are based on the official solution. When grading submissions that used different approaches, graders awarded points based on how close to (or far from) the student was from a complete (and correct) solution.

**1 point:** Student proposes using the inclusion-exclusion principle or another viable approach for solving this problem.

**1 point:** Student computes the probability that Zan removes the last of the 7 blue chips before the last of the 6 red chips.

**1 point:** Student computes the probability that Zan removes the last of the 7 blue chips before the last of the 8 green chips.

**1 point:** Student computes the probability that Zan removes the last of the 7 blue chips before the last of the 14 non-blue chips.

**1 point:** Student obtains the correct answer of $\frac{64}{195}$, with or without explanation.

**Problem 4/2/30:**

**1 point:** Student proposes treating both sides of the given equation as polynomials in $n$ or another viable approach to this problem.

**1 point:** Student rules out $a = 1$ or $b = 1$. To get full credit (**5 points**), the student must rule out both $a = 1$ and $b = 1$.

**2 points:** Student rules out $a > 2$ and $b > 2$. Award **1 point** of partial credit for significant constructive progress towards this result (i.e., the student obtains a useful intermediate result). Expanding both sides of the given equation is not sufficient to obtain this point, unless the student solution receives a total of **1 point**.

**1 point:** Student shows that $(a, b) = (2, 2)$ satisfies the conditions in the problem.

**Problem 5/2/30:**

**Note:** Student solutions varied considerably. The rubric below is based on the official solution, but in most cases we used the following more general guidelines: **1 point** for a diagram, **1 additional point** for a potentially useful result, and a total of **5 points** for a correct solution using an alternate method. In some cases, graders awarded **3 points** or **4 points**. In these cases, the student had a viable approach with significant constructive progress, but the solution was incomplete (typically **3 points**), or the student had a solution that was nearly complete and correct, but a key step wasn't fully justified (typically **4 points**).

**Note:** We appreciate that many students invested considerable time in creating helpful diagrams. Graders typically deducted **1 point** if a student did not include a diagram and the solution was hard to follow as a result.

**1 point:** Student draws a reasonable diagram that incorporates the given information.

**1 point:** Student shows that $R$ lies on the radical axis of the circumcircles of triangles $BXD$ and $CYD$.

**1 point:** Student shows that $H$ must lie on the circumcircle of triangle $BCR$, with line $XDY$ as the Simson line of point $H$ with respect to triangle $BCR$.

**1 point:** Student shows that triangles $ABC$ and $RBC$ are congruent.

**1 point:** Student completes the proof that $\overline{OH}$ is parallel to $\overline{BC}$ (e.g., by using Power of a Point).